14/3/2016 الاثنين د. عزت محاضرة $\boxed{5}$

in finding range of k problems, use Jury Test for 2nd order, and routh for higher orders

Report: $$\overline{GH}(z) = \frac{k(z-0.2)}{(z-1)(z+0.6)^2}$$

Find the range of k for stability using
- bilinear method (routh)
- Jury Test لو تجاهلت تأثير الصفر هتطلع معاك غلط

* Root Locus

Stability

① Relative Stability
بنعرف منها مدى استقرار النظام
(GM, PM are stability indicators)
- Bode Diagram
- polar plot
- Nyquist

② absolute Stability
- routh (bilinear transf.)
- Jury test

بنعرف منها حالة النظام
stable
unstable
critically stable

Stability

① Graphical Methods
- root locus
- Bode Diagram
- polar plot
- Nyquist

② Algebric Methods
- Jury test
- Routh array (using bilinear transformation)

Ex 1: $\overline{GH}(z) = \dfrac{K(z+1)}{(1-z)^2} \Rightarrow (1-z)^2 = (z-1)^2$

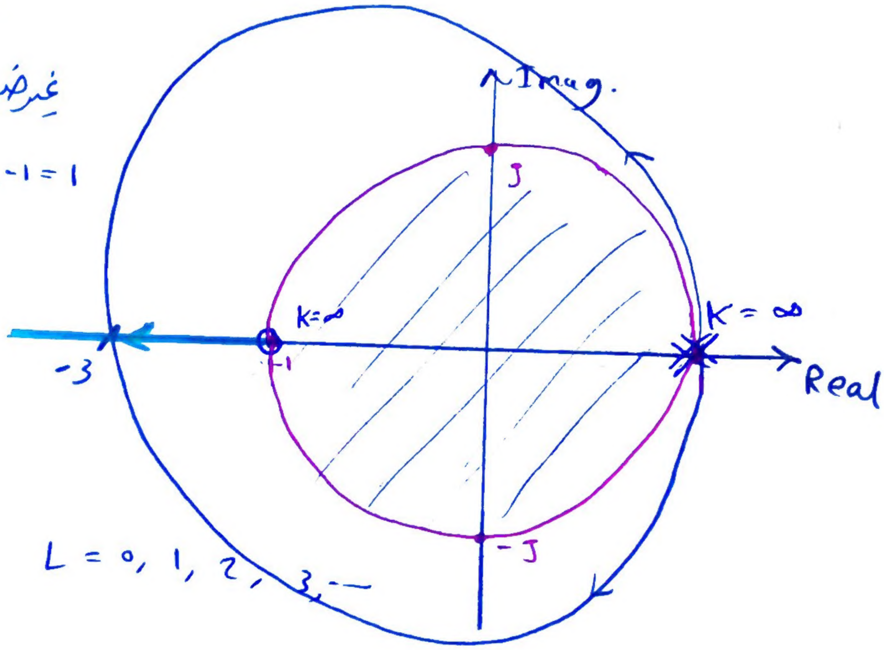
- Draw the root Locus and find the critical value for K (K_{cr})

* الفرق هنا هشوف مركز الـ poles بالنسبة لدايرة الوحدة

① Poles : $n_p = 2 \rightarrow 1, 1$

Zeros : $n_z = 1 \rightarrow -1$

② Z - plane



③ real part $\Rightarrow -1 : \infty$

④ Asymptotes غير ضرورية للمسألة هنا

1- no. of asy. $= n_p - n_z = 2 - 1 = 1$

2- $\sigma_c = \dfrac{\Sigma poles - \Sigma zeros}{n_p - n_z}$

$= \dfrac{1 + 1 - (-1)}{1}$

$= 3$

3- $\theta = \dfrac{(2L+1)\,180}{\cancel{n_p - n_z}\ 1}$ $L = 0, 1, 2, 3, \ldots$

⑤ Breaking points :-

- Ch. eq. $1 + \overline{GH}(z) = 0 \Rightarrow 1 + K\dfrac{(z-1)}{(z-1)^2}$

- $\dfrac{K(z+1)}{(z-1)^2} = -1 \Rightarrow K = -\dfrac{(z-1)^2}{(z+1)}$

- $\dfrac{dK}{dz} = 0 \Rightarrow \dfrac{dK}{dz} = -\left[\dfrac{(z+1)(2z-2) - (z-1)^2(1)}{(z+1)^2}\right] = 0$

$(z+1)(2z-2) - (z^2 - 2z + 1) = 0$

$z^2 + 2z - 3 = 0 \Rightarrow (z-1)(z+3) = 0$

$z = 1, -3$

\# Breaking points : ① at $z = 1 \Rightarrow$ Break away

at $z = -3 \Rightarrow$ Break in

K at $z=-3$ $= \dfrac{-(-3-1)^2}{(-3+1)} = 8$

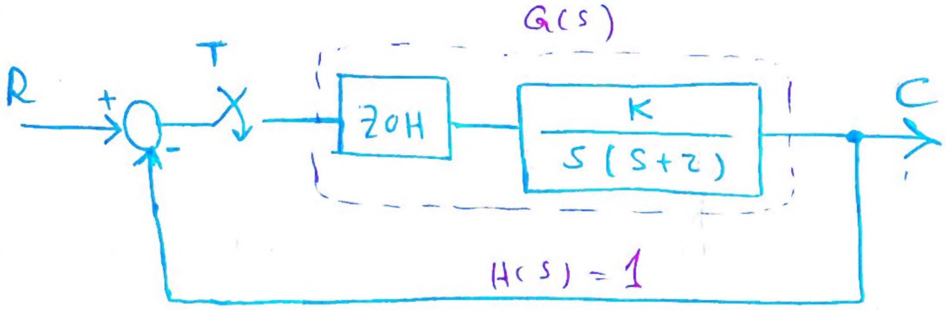
$r = \dfrac{1-(-3)}{2} = 2 \qquad C = 1 - r = 1-2 = -1$

r = radius of circle, C = center of circle

الحركة خارج الدائرة وبالتالي النظام unstable. لجميع قيم K

The System is unstable for all $K > 0$

Ex:



Draw the root locus for the above system for $T=0.4$ & $T=3$ and find K_{cr}

$$O.L.T.F = \overline{GH}(z) = G(z) = Z\left[\frac{(1-e^{-Ts})K}{s^2(s+2)}\right]$$

$$= (1-z^{-1})K\, Z\left[\frac{1}{s^2(s+2)}\right]$$

$$= K\,(1-z^{-1})\, Z\left[\frac{A_1}{s^2} + \frac{A_2}{s} + \frac{A_3}{s+2}\right] \qquad A_1 = 1/2,\quad A_3 = 1/4,\quad A_2 = -1/4$$

$$\overline{GH}(z) = K\left(\frac{z-1}{z}\right) Z\left[\frac{0.5}{s^2} - \frac{0.25}{s} + \frac{0.25}{s+2}\right]$$

$$= \frac{K}{4}\left(\frac{z-1}{z}\right) Z\left[2t - 1 + e^{-2t}\right] \qquad t = kT$$

$$= \frac{K}{4}\left(\frac{z-1}{\cancel{z}}\right)\left[\frac{2T\cancel{z}}{(z-1)^2} - \frac{\cancel{z}}{z-1} + \frac{\cancel{z}}{z-e^{-2T}}\right]$$

$$= \frac{K}{4}\left[\frac{2T}{(z-1)} - 1 + \frac{z-1}{z-e^{-2T}}\right]$$

$$= \frac{K}{4}\left[\frac{2T(z-e^{-2T}) - (z-1)(z-e^{-2T}) + (z-1)^2}{(z-1)(z-e^{-2T})}\right]$$

$$= \frac{K}{4}\left[\frac{(2T-1+e^{-2T})\,z + (1-2Te^{-2T}-e^{-2T})}{(z-1)(z-e^{-2T})}\right]$$

at $T = 3$ sec $\Rightarrow e^{-2T} = e^{-6} = 0.0025 \simeq 0$

$$\overline{GH}(z) = \frac{K}{4} * \frac{5z+1}{z(z-1)}$$

$$= \frac{5}{4}K\,\frac{z+0.2}{z(z-1)} = \underbrace{1.25K}_{\to K'}\,\frac{z+0.2}{z(z-1)}$$

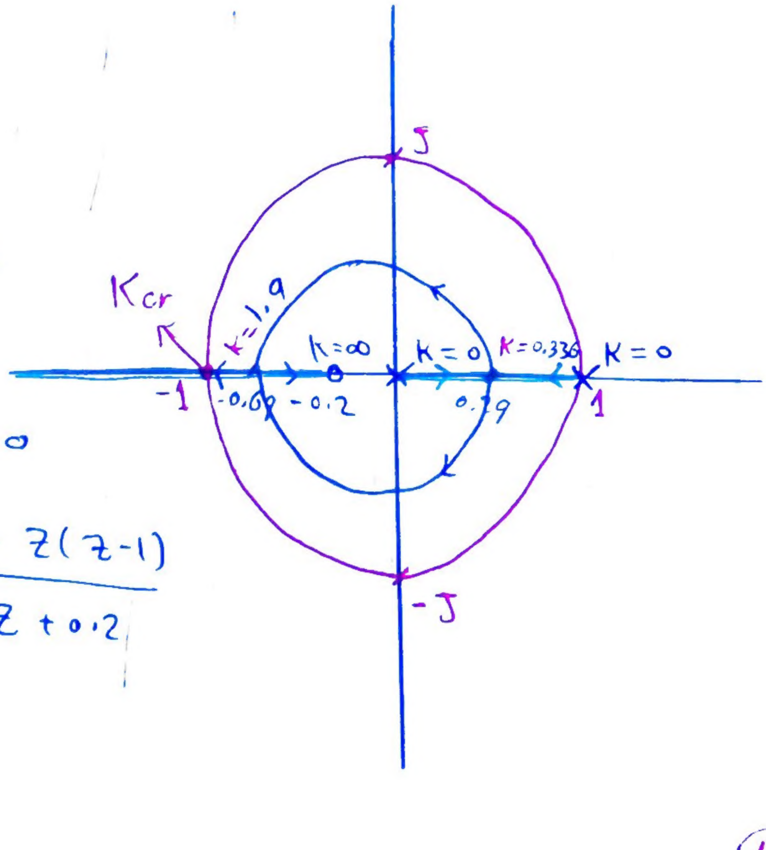
لازم معامل K بواحد
عوضت بـ K'
حيث $K' = 1.25K$

root locus:-

① poles: $n_p = 2 \longrightarrow 0, 1$

zeros: $n_z = 1 \longrightarrow -0.2$

② z-plane



③ real part $\rightarrow$ $0 : 1$

$-0.2 : -\infty$

④ Asymptotes $\longrightarrow$ no need for it

⑤ Breaking points

ch. eqn $\Rightarrow 1 + \overline{GH}(z) = 0$

$$1 + \frac{K'(z+0.2)}{z(z-1)} \Rightarrow K' = \frac{-z(z-1)}{z+0.2}$$

$$\frac{dK'}{dz} = 0 \Rightarrow -\frac{(z+0.2)(2z-1) - z(z-1)(1)}{(z+0.2)^2} = 0$$

$$(z+0.2)(2z-1) - z^2 + z = 0$$

$$z^2 + 0.4z - 0.2 = 0 \Rightarrow z_{1,2} = 0.29 \,, -0.69$$

Breaking points → ① Breakaway at $z_1 = 0.29$

$$K' = -\left[\frac{z(z-1)}{z+0.2}\right]\Bigg|_{z=0.29} = 0.42 = 1.25K$$

$$K\Big|_{z=0.29} = 0.336$$

→ ② Breakin at $z_2 = -0.69$

$$K'\Big|_{z=-0.69} = -\left[\frac{z(z-1)}{z+0.2}\right]_{z=-0.69} = 2.38 = 1.25K$$

$$K = \frac{2.38}{1.25} = 1.9$$

$r = 0.49$ $\quad c = -0.2$

system is stable for $0 < K < K_{cr}$

K_{cr} بنجيبها بالطريقة القديمة بنضرب أطوال الـ poles من النقطة K_{cr}
ونقسم على مضروب أطوال الـ zeros

⟹ continue

$\boxed{6}$ Determine K_{cr}

① $K' = \dfrac{\pi \text{ poles}}{\pi \text{ zeros}} = \dfrac{L_{P_1} \cdot L_{P_2}}{L_Z}$

$$K' = \frac{1 * 2}{0.8} = 2.5$$

$$K' = 1.25\, K \Rightarrow K = \frac{2.5}{1.25} = 2$$

$\boxed{K_{cr} = 2}$ $\Rightarrow$ range for Stability $0 < K < 2$

② Another method

$$K' = -\left[\frac{z\,(z-1)}{z+0.2}\right]$$

, The critical gain K_{cr} at $z = -1$

$$K'_{cr} = -\left[\frac{-(-1-1)}{-1+0.2}\right] = 2.5 \Rightarrow K_{cr} = 2$$

$\boxed{2}$ at $T = 0.4$ sec

$$\overline{GH}(z) = \frac{K}{4}\left[\frac{0.25\,z + 0.19}{(z-1)(z-0.45)}\right]$$

$$= \frac{K}{16}\left[\frac{z + \frac{0.19}{0.25}}{(z-1)(z-0.45)}\right]$$

$$= \frac{K}{16}\left[\frac{z + 0.76}{(z-1)(z-0.45)}\right]$$

$$= K'\left[\frac{z + 0.76}{(z-1)(z-0.45)}\right] \; ; \; K' = \frac{K}{16}$$

① Poles : $n_p = 2 \Longrightarrow 0.45, 1$

Zeros : $n_z = 1 \Longrightarrow -0.76$

② z-plane

$r = \frac{0.7 + 2.22}{2} = 1.46$; $c = 0.7 - 1.46 = -0.76$

③ Asymptotes $\longrightarrow$ no need

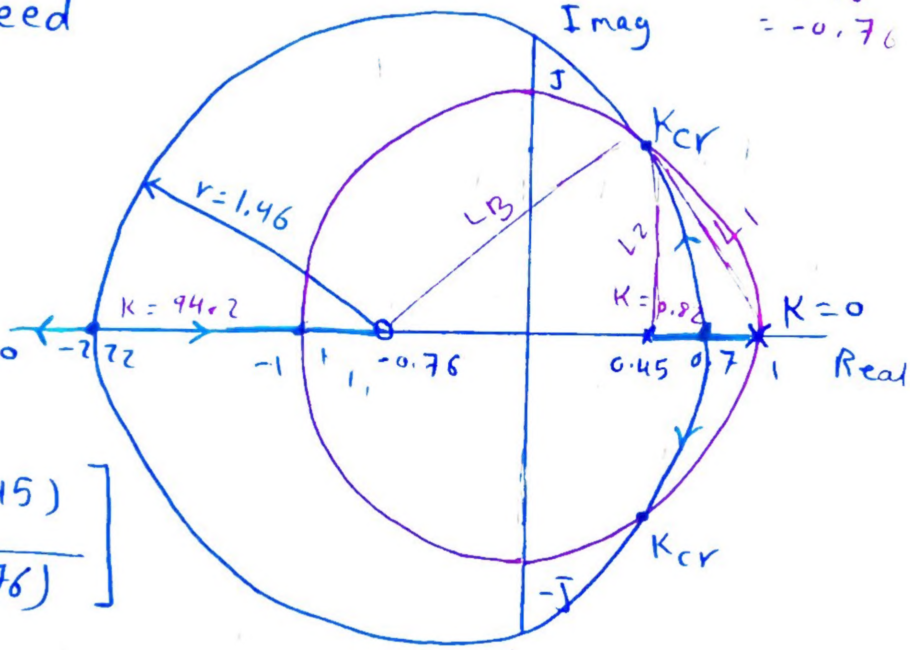
④ Breaking points

ch. eqn

$1 + \overline{GH}(z) = 0$

$$1 + K' \frac{(z+0.76)}{(z-1)(z-0.45)} = 0$$

$$K' = -\left[\frac{(z-1)(z-0.45)}{(z+0.76)}\right]$$



$$\frac{dK'}{dz} = 0 \Rightarrow z^2 + 1.52z - 1.552 = 0$$

$$z_{1,2} = 0.7 \; \& \; -2.22$$

Breaking points:

① Break away at $z_1 = 0.7$

$$K' = -\left[\frac{(z-1)(z-0.45)}{(z+0.76)}\right]_{z=0.7} = 0.05137 = \frac{K}{16}$$

$K = 16 * 0.05137 = 0.822$

② Breakin at $z_2 = -2.22$

$$K' = -\left[\frac{(z-1)(z-0.45)}{(z+0.76)}\right]_{z=-2.22} = 5.888 = \frac{K}{16}$$

$K = 16 * 5.888 = 94.2$

* $K'_{cr} = \frac{L_1 \; L_2}{L_3}$

$\boxed{6}$ Determine K_{cr}

$\boxed{1}$ $K'_{cr} = \frac{L_1 \; L_2}{L_3}$ ✓

$K = 16 K' = $ ✓

$\boxed{2}$ Using Jury test

ch. eqn $1 + GH(z) = 0$

$$1 + \frac{K'(z+0.76)}{(z-1)(z-0.45)} = 0 \Rightarrow (z-1)(z-0.45) + K'(z+0.76) = 0$$

$$F(z) = z^2 + (-1.45 + K')z + (0.45 + 0.76K') = 0$$

$$0 < K' < 0.7237$$

$$0 < \frac{K}{16} < 0.7237 \Rightarrow 0 < K < 11.578$$

$K_{cr} = 11.578$

$\boxed{3}$ Get the intersection of the two circles

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

Center $= (x_0, y_0)$

r = radius

① unit circle

$$\boxed{x^2 + y^2 = 1}$$

② r-locus circle

$$(x + 0.76)^2 + y^2 = (1.46)^2$$

حل المعادلتين مع بعض هات قيم

(x, y)

$$\cancel{x^2} + 1.52x + 0.5776 + (1 - \cancel{x^2}) = 2.1316$$

$x = 0.364$

$y^2 = 1 - (0.364)^2$

$y = \pm j0.931$

The critical gain K'_{cr} at

$z = 0.364 \pm j0.931$

عوض في معادلة الـ K' عشان تجيب قيمتها

$$K' = -\frac{(z-1)(z-0.45)}{(z+0.76)} \Rightarrow |K'| = \left. \frac{|z-1|\,|z-0.45|}{|z+0.76|} \right|_{z=0.364+j0.931}$$

$$\boxed{K' = 0.7223}$$

$K = K' * 16 = 11.56$